temperature distribution along the system when the Thomson heat is either released or absorbed, K; $U^{(I)}$, $U^{(II)}$, voltage drops for two current directions, V; σ, Stefan–Boltzmann constant, $W/(m^2 \cdot K^4)$; σ_{eff}, effective Thomson coefficient for the "tube–metal" system, V/K; ε_t, emissivity of the tube. Superscripts: t, tube; subscripts: eff, effective; ψ, ψl, current magnitudes; c, cross-sectional area; s, side surface.

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FORMATION OF A LAYER OF A LIQUID AS IT IMPINGES ON A HORIZONTAL PLANE

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UDC 532.62

We have conducted a numerical study of the spreading out of a liquid over a horizontal plane, with the liquid, in this case, running off over the surface of a semiinfinite vertical cylinder.

When a liquid impinges on a horizontal surface it spreads out and as a result a liquid layer of a specific thickness is formed on the surface. A characteristic unique feature of the flow achieved in this case is the presence of a free surface. The flow of a viscous liquid over a horizontal surface with a relatively small layer thickness has been studied in a number of papers [1-6]. Attempts have been made numerically to solve the problem of the spreading out of a column of liquid under the force of gravity [7-9]. In this particular study we examine the axisymmetric motion of a viscous liquid over a horizontal plane, with the liquid, in this case, running off over the surface of a semiinfinite vertical cylindrical rod, impinging on a horizontal plane. The motion is assumed to be creeping, so that the inertial forces may be regarded as negligibly small in comparison to the viscosity forces. The capillary forces are assumed to be small in comparison to the viscosity and gravitational forces, and thus are also not taken into consideration.

1. Formulation of the Problem. In a cylindrical coordinate system the system of equations describing the flow, in conjunction with the above assumptions, has the form

$$\mu\Delta u - \frac{\partial p}{\partial z} - \rho g = 0, \quad \mu\left(\Delta v - \frac{v}{r^2}\right) - \frac{\partial p}{\partial r} = 0, \quad \Delta p = 0. \tag{1}$$

The third of the equations in (1) is a consequence of the first two and of the condition of incompressibility.

The conditions specifying an absence of tangential stress, equality of the normal stress to the external pressure, and the kinematic condition, are all satisfied at the free surface:

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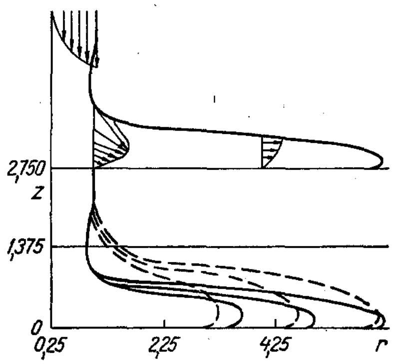

Fig. 1

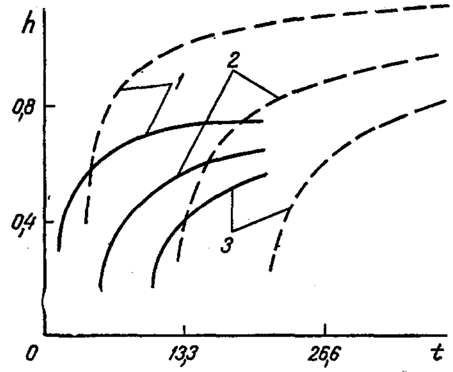

Fig. 2

Fig. 1. Structure of the flow and the evolution of the free surface in the liquid spreading process.

Fig. 2. Change in the thickness of the layer over time.

$$s\Pi n = 0, \quad n\Pi n = -p_0, \quad \frac{dz}{dt} = u, \quad \frac{dr}{dt} = v. \tag{2}$$

The condition of adhesion is utilized at the surface of the cylindrical rod and on the horizontal plane:

$$u = v = 0, \quad z = 0, \quad R_1 \leqslant r \leqslant R(t); \quad 0 \leqslant z < \infty, \quad r = R_1. \tag{3}$$

At some distance from the horizontal plane, the liquid impinges at a constant rate of flow Q, with the velocity profile in this case coincident with the solution of the problem dealing with the steady-state flow in a liquid layer of constant thickness, running off along the vertical cylindrical surface [10]. Thus, the boundary conditions at infinity have the form

$$u = a\left(r^2 - R_1^2 - 2\ln\frac{r}{R_1}\right), \ v = 0 \ (a > 0), \ Q = \text{const}, \qquad z \to \infty, \quad R_1 \leqslant r \leqslant R_0. \tag{4}$$

At the initial instant of time, the liquid layer is situated on the surface of the vertical cylindrical rod, and the free surface is formed by a cylindrical surface of radius R_0. In selecting R_0 it is essential that we bear in mind that the thickness of the liquid layer in the case of a stabilized flow is determined by the relationship between the viscosity and gravitational forces in conjunction with the rod radius.

In order to achieve equivalence in the solution of the problems involving the utilization of Eqs. (1) relative to the solution of the problem involving the continuity equation, in the place of $\Delta p = 0$ the following condition of incompressibility at the boundaries must be satisfied [11]:

$$\nabla V = 0. \tag{5}$$

2. Calculation Method. The solution of the stated problem is found by a difference method in the end region; in this case, the boundary for which conditions (4) have been satisfied must be sufficiently removed from the horizontal plane. The difference analogs of Eqs. (1) are written in a grid which contains irregular nodes in the vicinity of the free surface. For the solution we resort to the Gauss–Seidel iteration method. The first of the conditions in (2), in combination with the incompressibility condition (5) at the free surface, is written in a local orthogonal coordinate system in the form proposed in [12], which makes it possible to use a moving calculation scheme to calculate the velocity-vector components. For more precise satisfaction of the difference analog of the incompressibility condition within the calculation region, a correcting procedure is employed in each iteration [13, 14], which involves the introduction of a correction velocity potential. Use of this procedure to recalculate the velocities derived from the difference analogs of the first two equations in (1) ensures the solenoidality of the velocity vector field over the entire duration of the flow without accumulation of error.

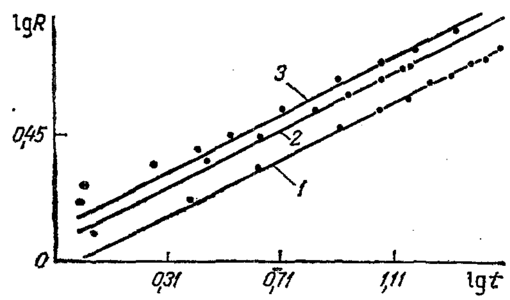

Fig. 3

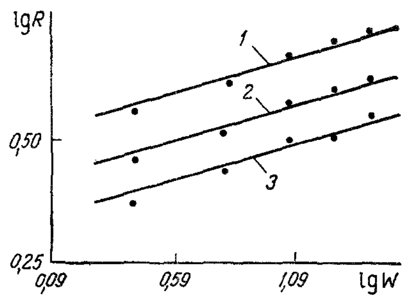

Fig. 4

Fig. 3. Functions characterizing the change in the radius of the layer over time.

Fig. 4. The radius of the layer as a function of W.

The general sequence of calculation involves the following.

The solution of the difference analogs of Eqs. (1) with satisfaction of conditions (2)-(5) at the boundaries is found by means of the iteration schemes. The pressure values at the surface of the cylindrical rod are found from the difference analog of the second of the equations in (1), while those values at the horizontal plane are found from the difference analog of the first of the equations from (1). The pressure at the free surface is found from the second condition in (2). In calculating the velocity-vector components in each iteration the velocities are recalculated by means of a correcting procedure [14]. After obtaining the steady-state velocity and pressure fields from the difference analogs of the last two conditions in (2) we determined a new position and shape for the free surface. In this derived region the solution for system of equations (1) is obtained once again. Thus, we achieve a sequence of quasisteady solutions and the change in the free surface with the passage of time.

3. Study Results. In discussing the results, let us use the dimensionless complex $W = \rho g R_0{}^2/\mu u_0$, characterizing the ratio of the gravitational and viscosity forces. Here $u_0 = Q/\pi(R_0{}^2 - R_1{}^2)$ is the average translational velocity of the liquid as $z \to \infty$.

With the exception of W, this flow is determined by the dimensionless radius $\bar{R}_1 = R_1/R_0$ of the cylindrical rod, with the bar in the notation of the dimensionless quantities dropped in the following. Figure 1 shows the evolution of the free surface in the runoff process for various values of W when $R_1 = 0.25$, with the solid curves representing W = 24.7 (t = 7, 13, 21) and the dashed curves W = 2.5 (t = 11, 24, 40). The structure of the flow at the instant of time at which the horizontal plane gives rise to the formation of the liquid layer is shown in the upper portion of Fig. 1. We can divide the entire flow region into three zones. The first zone covers the flow in the layer of the liquid on a vertical wall with the characteristic velocity profile (4), while the second zone covers the transition of vertical flow to horizontal; finally, the third zone represents the flow within the liquid layer on the horizontal plane.

Figure 2 shows the change in the liquid layer thickness h on the horizontal plane over time t, referred to the quantity R_0/u_0, in various sections r = const, $R_1 = 0.25$: curves 1) r = 2; 2) 3.5; 3) 4.5. The solid curves are a result of calculations conducted at W = 24.7; the dashed curves represent calculations conducted for W = 2.5, $R_1 = 0.25$. The layer formed on the horizontal surface exhibits a wedge-shaped form and the thickness of the layer, all other conditions being equal, increases as W is reduced.

An approximate nonlinear equation is presented in [5] for the thickness of the layer formed in the spreading out of a viscous liquid over a horizontal plane under the force of gravity:

$$h_t = \frac{g}{3\nu}\nabla(h^3\nabla h). \qquad (6)$$

The self-similar solution of Eq. (6) for the case of the spreading out of an axisymmetric spot in whose center a constant liquid flow of intensity Q is maintained yields an expression for the spot radius in the form [5, 15]:

$$R = AQ^{3/8}\left(\frac{g}{\nu}\right)^{1/8} t^{1/2}, \tag{7}$$

where A is a dimensionless universal constant equal to 0.65 according to the experimental data of [4].

In the flow-spreading stage, when the diameter of the liquid layer exceeds its thickness, the influence of the flow in the transition zone and the presence of a solid side wall on the motion of the leading front of the layer ceases to be significant, and the problem of describing the spreading process can be formulated with utilization of Eq. (6) in analogy with [15]. Thus, assuming satisfaction of relationship (7), in the case under consideration, given a relatively small layer thickness, we can write the dimensionless analog of (7) in the form

$$R = BW^{1/8}t^{1/2}, \quad B = A(1 - R_1^2)^{3/8}\pi^{3/8}. \tag{8}$$

The relationships which characterize the change in the radius of the layer over time for various W are shown in Fig. 3: 1) W = 2.5; 2) 12.3; 3) 24.7 (R_1 = 0.25). The points represent the results of the calculations, while the solid lines are straight, with a slope of 0.5. The calculation results confirm the law $R \sim t^{1/2}$ and make it possible to determine the time from which this law may take effect. Deviation of the points at the initial instant of time is a consequence of the significant non-one-dimensionality of the flow in the zone of transition from vertical to horizontal flow, the influence of the side wall, and the failure to satisfy the condition of smallness for the relative thickness of the layer, i.e., failure to fulfill the basic assumptions under which the solution for (7) was obtained. In order to confirm the validity of the solution of (8) over rather long periods of time, we have to examine the relationship between R and W. The points in Fig. 4 show the calculation results which characterize the radius of the layer as a function of W, and with R_1 = 0.25 the solid lines are straight, with a slope of 0.125: 1) t = 13.4; 2) 8.1; 3) 5.7. The results of the calculation satisfactorily confirm the law $R \sim W^{1/8}$.

The results of the calculations carried out for R_1 = 0.625 and $2.5 \le W \le 25$ also confirm the validity of the solution for (8) for prolonged periods of time. The difference between the values of B obtained by calculation and from expression (8) does not exceed 10%.

NOTATION

z, r, coordinates in the cylindrical coordinate system; u, v, velocity components in the z and r directions, respectively; μ, dynamic viscosity; p, pressure; s, n, unit vectors, tangential and normal to the free surface; Π, stress tensor; p_0, external pressure; t, time; R_1, radius of cylindrical rod; R_0, radius of liquid layer on cylindrical rod at initial instant of time; V, velocity vector; R(t), radius of liquid layer on horizontal plane; g, gravitational acceleration; ρ, density.

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FLOW OF AN ANOMALOUS VISCOUS FLUID IN A CENTRIFUGAL JET NOZZLE

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The motion of twisted flows of an anomalous viscous fluid subject to an exponential law has been studied numerically.

Centrifugal jet nozzles have found extensive application in chemical technology apparatus requiring uniform spraying densities (absorbers, wet dust collectors, hydraulic foam extinguishers). In this case, the spray medium is generally a form of an anomalous viscous fluid: solutions of surface-active materials, suspensions, etc.

Let us take a look at the flow of an anomalous viscous fluid subject to an exponential law in a centrifugal jet nozzle (Fig. 1). We will assume the regime of motion to be both steady-state and axisymmetric. We will separate the flow region into the following zones:

I) the peripheral flow, bounded by the conical surfaces of frame 1 and insert 2;

II) the central flow in the channel, with a threaded insert;

III) the zone in which the peripheral and central flows are mixed.

I. We will examine the motion of the fluid in a special orthogonal curvilinear coordinate system ℓ, φ, δ, with the ℓ axis coincident with the generatrix of the internal cone. We will assume in the solution that 1) the influence of mass forces on the flow of the fluid is negligibly small; 2) that the velocity in the direction of the δ axis is considerably smaller than the velocity in the direction of the ℓ axis. The system of differential equations of fluid motion with consideration of [1] will then assume the following form:

$$\rho V_l \frac{\partial V_l}{\partial l} - \frac{\rho V_\varphi^2 \sin\alpha}{\delta\cos\alpha - l\sin\alpha} = -\frac{\partial p}{\partial l} + K\frac{\partial}{\partial\delta}\left(E^{n-1}\frac{\partial V_l}{\partial\delta}\right) + \frac{KE^{n-1}\cos\alpha}{\delta\cos\alpha - l\sin\alpha}\frac{\partial V_l}{\partial\delta},$$

$$\rho V_l \frac{\partial V_\varphi}{\partial l} - \frac{\rho V_\varphi V_l \sin\alpha}{\delta\cos\alpha - l\sin\alpha} = K\frac{\partial}{\partial\delta}\left(E^{n-1}\frac{\partial V_\varphi}{\partial\delta}\right) + \frac{2KE^{n-1}\cos\alpha}{\delta\cos\alpha - l\sin\alpha}\frac{\partial V_\varphi}{\partial\delta}, \qquad (1)$$

$$-\frac{\rho V_\varphi^2 \cos\alpha}{\delta\cos\alpha - l\sin\alpha} = -\frac{\partial p}{\partial\delta} + K\frac{\partial}{\partial l}\left(E^{n-1}\frac{\partial V_l}{\partial\delta}\right) - \frac{KE^{n-1}\sin\alpha}{\delta\cos\alpha - l\sin\alpha}\frac{\partial V_l}{\partial\delta}.$$

Here

$$E = \sqrt{\left(\frac{\partial V_l}{\partial\delta}\right)^2 + \left(\frac{\partial V_\varphi}{\partial\delta}\right)^2}.$$

The boundary conditions will be as follows:

$$V_l = V_\varphi = 0 \text{ for } \delta = 0; \quad \delta = \delta_0. \qquad (2)$$

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